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Airfoil Design by Numerical Optimization Using a Minicomputer

Raymond M. Hicks and C. A. Szelazek

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Raymond M. Hicks
Ames Research Center
Moffett Field, California

and

C. A. Szelazek
Computer Information Systems
Cupertino, California

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and Space Administration

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NOMENCLATURE

A cross-sectional area of airfoil

c chord

c_l section lift coefficient

c_m section pitching moment referenced to quarter chord

c_p pressure coefficient $\frac{p_1 - p_\infty}{q_\infty}$

p_1 local static pressure, N/m^2

p_∞ free-stream static pressure, N/m^2

q_∞ free-stream dynamic pressure, N/m^2

r_0 distance from origin to center of auxiliary circle, m

r_1 distance from origin to center of main circle, m

t airfoil thickness, m

w weighting parameter

x horizontal axis

y vertical axis

α angle of attack, deg

θ angle between x-axis and r_1, deg

ρ radius of auxiliary circle, m

ϕ angular orientation of auxiliary circle, deg

ψ angle between x-axis and r_0, deg

AIRFOIL DESIGN BY NUMERICAL OPTIMIZATION

USING A MINICOMPUTER

Raymond M Hicks
Ames Research Center

and

C. A Szelazek
Computer Information Systems

SUMMARY

An airfoil design program has been developed for the automated design on a minicomputer of low speed airfoils The program utilizes a generalized Joukowski method for aerodynamic analysis coupled with a conjugate gradient, penalty function, numerical optimization algorithm to give an efficient calculation technique for use with minicomputers.

The program has been used to design airfoils with a prescribed pressure distribution and to develop airfoils which minimize or maximize some aerodynamic force coefficient At present the method is restricted to inviscid, incompressible flow.

A typical design problem will execute in 4.5 hr on an HP 9830 minicomputer.

INTRODUCTION

A major effort has been undertaken over the last 4 years to develop numerical optimization methods for use in airfoil section and wing design (refs 1–8) The techniques have been applied to design problems for low speed and transonic aircraft. Experimental evaluation of profiles and wings designed by the numerical optimization methods have proved successful (ref 9).[1] The computer codes used during the studies reported in references 1–8 were developed for a CDC 7600 computer, one of the most powerful scientific computers available today.

Because of the size of the codes described in the above cited references and the computer capacity required, the designer is sometimes faced with use of a code that is far more elaborate than needed for some design problems, particularly for low speed applications. Because many designers of light aircraft do not have the need for such large codes or the resources available to acquire powerful computers, an interest in automated airfoil design codes for minicomputers has developed. One such design program was reported in reference 10. The program described there is applicable to low speed design but does not have optimization capability

[1] Also in "Application of Numerical Optimization to the Design of Advanced Supercritical Airfoils," by Raymond R Johnson and Raymond Hicks Paper presented at the Langley Conference on Airfoil Design, March 1978

In the present effort the emphasis was placed on coupling a simple aerodynamic code with an efficient numerical optimization algorithm so that the final design code could be stored and executed on a relatively inexpensive minicomputer A generalized Joukowski method was selected as the aerodynamic analysis code because of its simplicity and an ability to generate a large class of airfoil contours The method is similar to that described in reference 11. The numerical optimization algorithm is based on a conjugate gradient technique combined with a penalty function approach to give an efficient, yet simple method for treating constrained optimization problems

The airfoil design code is presently applicable to incompressible, inviscid, two-dimensional flow problems and has been installed on an HP 9830 minicomputer In the near future, a simple boundary layer subroutine with a separation predictor will be added to the code to give the designer some limited capability in viscous flow design

DESIGN METHOD

The airfoil design program consists of a generalized Joukowski, aerodynamic analysis code[2] coupled with a conjugate gradient, penalty function, numerical optimization algorithm The program, written in BASIC, can be used with an HP 9830 minicomputer A photograph of the computer with plotter is shown in figure 1.

The aerodynamic analysis code is based on the following transformations

$$W = Z' + \frac{1}{Z'} \tag{1}$$

$$Z' = Z + \frac{B}{Z - Z_0} \tag{2}$$

where all quantities are complex Equation (1) is the well-known Joukowski transformation which transforms a circle into an airfoil if the center of the circle is displaced from the origin Let the center of the circle be given by

$$\mu = r_1 \, e^{i\theta_1} \tag{3}$$

Equation (2) transforms an exact circle to a distorted circle. Four parameters, which can be used as design variables for numerical optimization, are obtained by writing B and Z_0 in polar form

$$B = \rho \, e^{i\phi} \tag{4}$$

$$Z_0 = r_0 \, e^{i\psi} \tag{5}$$

Taking the four parameters ρ, ϕ, r_0, and ψ from equations (4) and (5) along with the two parameters r_1 and θ_1 from equation (3) the designer has six design variables available for numerical optimization.

[2] The generalized Joukowski method used during this study was suggested by R T Jones, Senior Staff Scientist at Ames Research Center

The most important parameters in determining the basic camber and thickness distributions of the airfoil are r_1 and θ_1. Examples will be presented to demonstrate the influence of each of the six parameters on the airfoil shape and to aid the reader in understanding the discussion of numerical optimization to be given later.

The baseline airfoil chosen for the parameter sensitivity study and the values of the parameters which determine the airfoil contour are shown in figure 2. The pressure distribution for $\alpha = 0°$ is also shown. The changes in airfoil shape resulting from perturbing each of the six parameters while leaving the remaining five unchanged are shown in figures 3–8. The baseline airfoil has been superimposed on each of the perturbed airfoils to clarify the effect of each parameter. Note that all parameters affect both thickness and camber of the baseline airfoil. It appears that the set of parameters available for design is sufficient to permit the designer to develop a relatively large class of airfoil contours through numerical optimization. The airfoil sections shown in figures 2–8 have a trailing edge thickness of 0 01 c which was achieved by adding parabolas, defined by $0.005(x/c)^{1/2}$, to the upper and lower surfaces of the airfoil The effect of such parabolas on the pressure distributions are not included in the Joukowski method but were shown to be negligible by an independent calculation. Hence, blunt trailing edges of as much as 0.01 c can be attained with the method proposed here with reliable pressure and force coefficients.

A schematic flow chart of the optimization process is shown in figure 9. The hypothetical design problem shown in the figure is pitching moment minimization. The first step in the design process is calculation of the aerodynamic coefficients of the initial airfoil section. In this case the initial profile was chosen to be the section shown in figure 2. The aerodynamic coefficients of the initial profile are stored as baseline values for use in the finite difference gradient calculations. The optimization program then perturbs each of the six design variables one at a time, returning to the aerodynamics code for evaluation of the aerodynamic coefficients and the partial derivative of pitching moment with respect to each design variable after each perturbation. The six different airfoil sections shown around the optimization loop of figure 9 were reproduced from figures 3–8 so that the effect of each design variable change on c_m and other aerodynamic and geometric parameters can be ascertained from the appropriate figure. The partial derivatives of pitching moment with respect to each design variable are calculated by one-sided finite difference. For example

$$\frac{\partial c_m}{\partial r_0} = \frac{-0\ 148 - (-0.131)}{-0\ 3 - (+0.3)} = \frac{-0\ 017}{-0\ 6} = 0\ 0283$$

This partial derivative along with the other five derivatives form the gradient of pitching moment (∇c_m) The direction in which the six variables are changed to reduce the nose-down pitching moment is $-\nabla c_m$ (the steepest descent direction) The optimization program increments the six design variables simultaneously in the direction indicated by $-\nabla c_m$. In this case the value of r_0 would be increased since an increase in r_0 was shown to decrease the nose-down pitching moment (compare figs. 2 and 3) Similarly ψ would be decreased, r_1 decreased, θ_1 increased, ρ decreased, and ϕ increased (figs 4 through 8). The process continues until the nose-down pitching moment starts to increase due to nonlinearity in the design space, at which time a new gradient is calculated and a new direction is determined which will again decrease the nose-down pitching moment. The optimization procedure continues until a local or global minimum is attained. When this occurs, the optimization process terminates and control is returned to the aerodynamics program for calculation of the final profile shape and the aerodynamic coefficients

The example shown in figure 9 is an unconstrained pitching-moment minimization problem. In actual design, such problems are of little practical value because the improvement in the objective function is usually accompanied by a degradation in some other aerodynamic or geometric property of the airfoil, e g , a reduction in nose-down pitching moment may be accompanied by a decrease in lift coefficient or airfoil thickness, both of which may be undesirable The airfoil design program developed during this study makes such undesirable changes avoidable by use of penalty functions For example nose-down pitching moment could be reduced without permitting corresponding reductions in lift coefficient and airfoil thickness by defining the objective function as

$$\mathrm{OBJ} = -|c_m| + W_1 c_l + W_2 (t/c)_{\max}$$

where W_1 and W_2 are weighting parameters adjusted during the optimization process to control reduction in the values of c_l and $(t/c)_{\max}$ The term, "penalty function," derives from the fact that the objective function is penalized (i.e., moves in a direction opposite to that which is desired) if c_l and $(t/c)_{\max}$ decrease

It is worth noting that the design variables used here do not permit the attainment of as large a class of airfoil contours as the shape functions described in reference 7 because of the lack of ability to achieve localized perturbations in airfoil contour As shown in figures 2–8 each of the six parameters used here affects changes over most of the profile. This means that the optimization algorithm must combine the six parameters carefully to achieve local modification of a profile. Local modification may be important for some design problems, e.g , leading edge modification to reduce pressure peaks

RESULTS AND DISCUSSION

The primary objective of this study was to demonstrate that it is possible to design good low-speed airfoil sections on a minicomputer. The following four design problems were considered during the demonstration. (1) design an airfoil to produce a specified pressure distribution, (2) unconstrained pitching-moment minimization, (3) pitching-moment minimization with a lift coefficient constraint, and (4) pitching-moment minimization with constraints on enclosed area and lift coefficient.

A problem which often confronts the airfoil designer is that of finding an airfoil that will give a desired pressure distribution Conventional inverse codes are often difficult to use and may be too large to store on minicomputers with limited core capacity. Hence, the first problem considered during the current study was to evaluate the usefulness of the numerical optimization code developed during this study as a pseudo-inverse method. The technique used here consists of defining the objective function as

$$\mathrm{OBJ} = \sum_{i=1}^{N} (c_{pd_i} - c_{p_i})^2$$

where c_{p_d} and c_p are the desired and actual pressure coefficients, respectively, and N is the number of pressure coefficients. The results of designing to a specified pressure distribution are shown in

figures 10 and 11 The initial profile, initial pressure distribution, and the desired pressure distribution are shown in figure 10 Note that the initial airfoil exhibits a peak negative pressure coefficient of -3.3 and a strong adverse pressure gradient following the peak. Such a pressure distribution is indicative of leading edge stall and low maximum lift coefficient The desired pressure distribution shown in figure 10 would clearly produce a better maximum lift coefficient than the initial distribution The final profile and the final pressure distribution are shown in figure 11 Note that the desired pressure distribution was nearly attained. The final profile exhibits slightly larger lift coefficient and nose-down pitching moment, and nearly twice the area of the initial profile

The results of an unconstrained pitching-moment minimization problem are shown in figure 12. Since no constraints were imposed on the design, the optimization algorithm took the path of least resistance, which was to make the airfoil more symmetrical Although this achieved the desired pitching moment reduction, the lift coefficient decreased substantially. Hence, the final airfoil may not be very useful

The results from imposing a lift coefficient constraint on the design are shown in figure 13, that is, the objective function becomes

$$\mathrm{OBJ} = -|c_m| + Wc_l$$

where the weighting parameter W was adjusted during the optimization process to prevent the lift coefficient from deviating more than 10% from the initial value Note that the reduction in nose-down pitching moment achieved here is less than that shown in figure 12 but still substantial considering that the value of c_l changed very little The final profile is thicker and has less aft camber than the initial section The reduction in aft camber accounts for the smaller nose-down pitching moment of the final profile Graphs of the values of pitching moment and lift coefficient vs iteration number are shown in figures 14(a)-(b) The oscillations observed in the figures are produced by the adjustments made in the weighting factor W during the optimization process The weighting factor was increased or decreased by 15% and a new gradient was calculated if the value of c_l decreased or increased by 10% from the initial value Decreasing the percent change in W from 15 to 5% had very little effect on the magnitude of the oscillations.

The final design problem considered during this study was pitching-moment minimization with upper and lower constraints on cross-sectional area and lift coefficient In this case both c_l and area were required to remain within 10% of the initial values The results of this optimization problem are shown in figure 15 Note that the final value of c_l is nearly at the lower constraint value whereas the cross-sectional area showed little change at the end of the design process There are two possible explanations for this result (1) the initial weighting factor chosen for c_l may have been too small, while the weighting factor used with area was correct, and (2) because of the oscillatory nature of c_m, c_l, and area, produced by the adjustable weighting factors, the final iteration may not produce the best result in the opinion of the designer To reconsider the case shown in figures 14(a)-(b), it is obvious that if the program stopped after the ninth iteration, c_m would have been reduced, but c_l would also have been somewhat lower At this stage the designer must decide which of the final iterations is preferable

For the final design problem the nose-down pitching moment decreased by nearly 30%, which is a smaller reduction than that achieved during the two preceding design problems (figs 12 and 13). As expected, the results of the three pitching moment minimization problems indicate that

the magnitude of the design improvement is a function of the number of constraints imposed on the design. More sophisticated numerical optimization algorithms will be developed for minicomputers in the future to eliminate some of the limitations of the penalty function method

CONCLUDING REMARKS

A practical, automated design technique for developing low-speed airfoil sections has been demonstrated. The technique can be used to design an airfoil to give a desired pressure distribution or to minimize or maximize aerodynamic force coefficients. The program is written in BASIC and will execute on a minicomputer with an 8K core.

Future work is needed to introduce a suitable boundary layer subroutine into the program so drag minimization and flow separation minimization can be achieved.

Further effort is also needed to replace or supplement the penalty function algorithm with a feasible direction-type algorithm capable of treating multiple inequality constraints.

The execution time required for a design problem using five optimization iterations is 4.5 hr on an HP 9830 computer or 0.5 hr on an HP 9845 computer. An HP 9845 computer with plotter costs about $30,000 in 1978 dollars

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif 94035, June 21, 1978

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Figure 1.– Minicomputer with plotter.

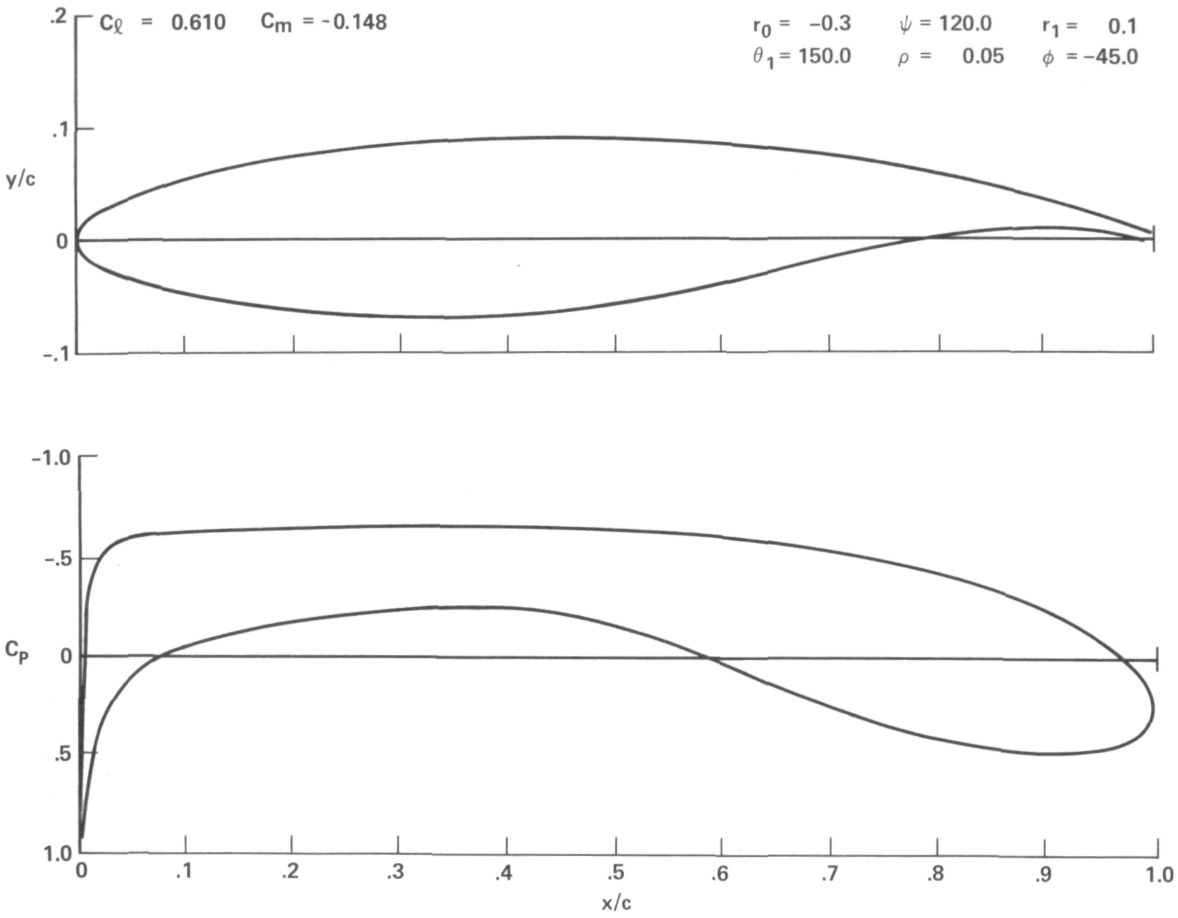


Figure 2.– Baseline airfoil and pressure distribution, $\alpha = 0°$.

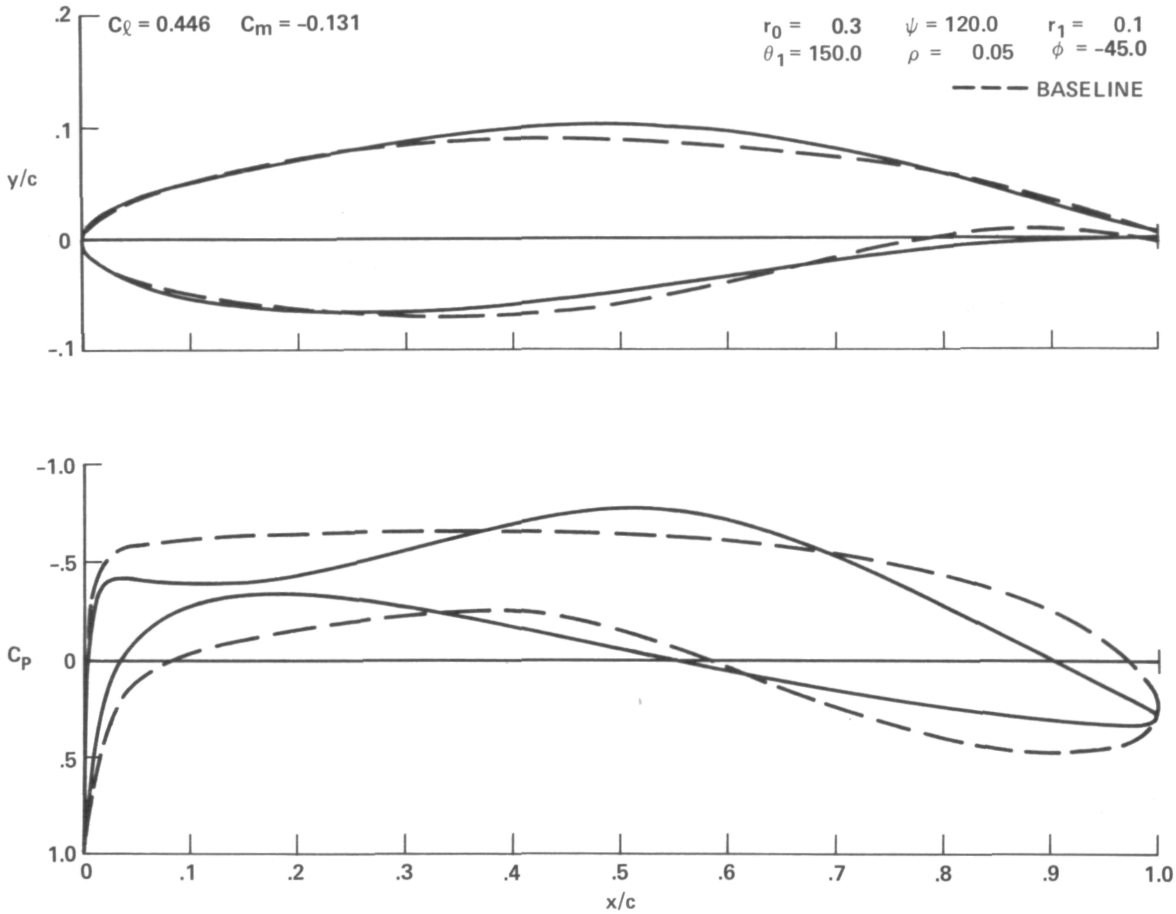


Figure 3.– Profile and aerodynamic changes due to r_0 perturbation, $\alpha = 0°$.

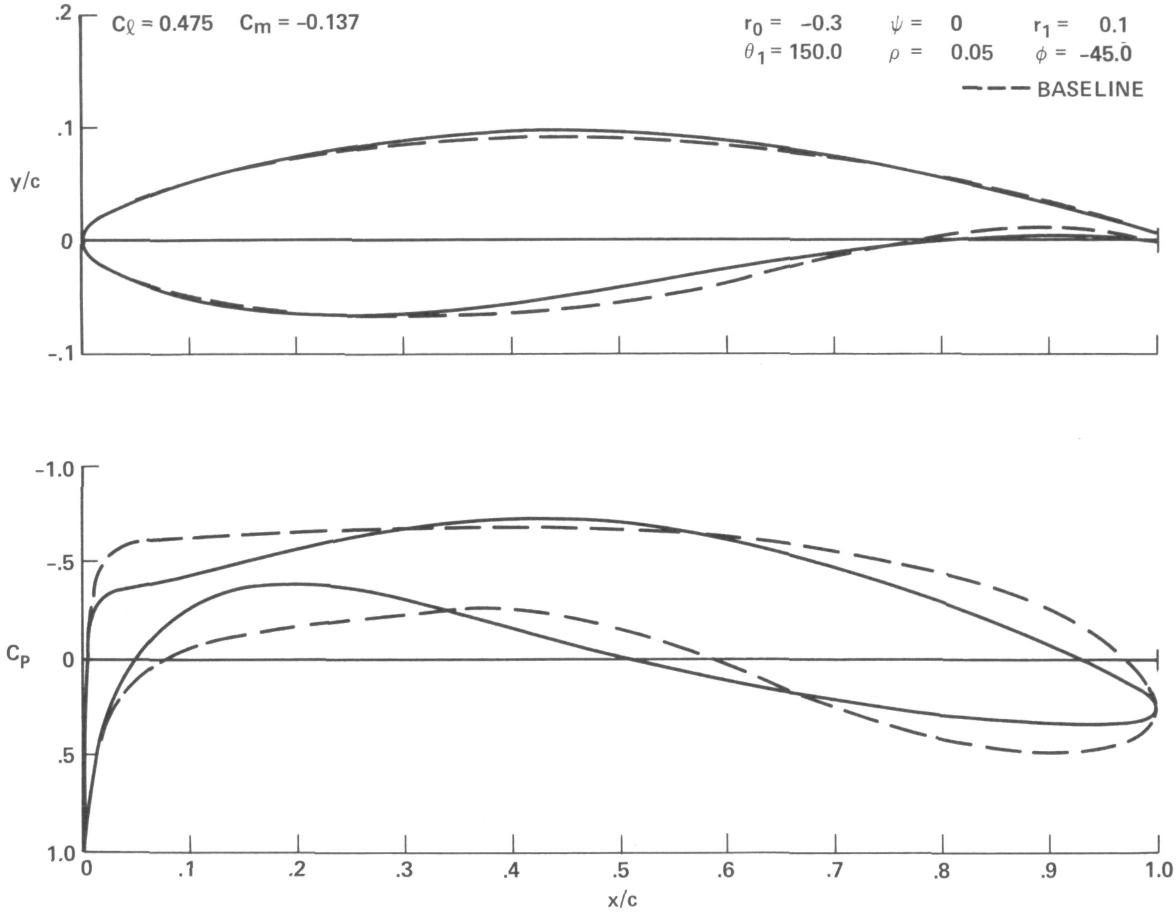


Figure 4.– Profile and aerodynamic changes due to ψ perturbation, $\alpha = 0°$.

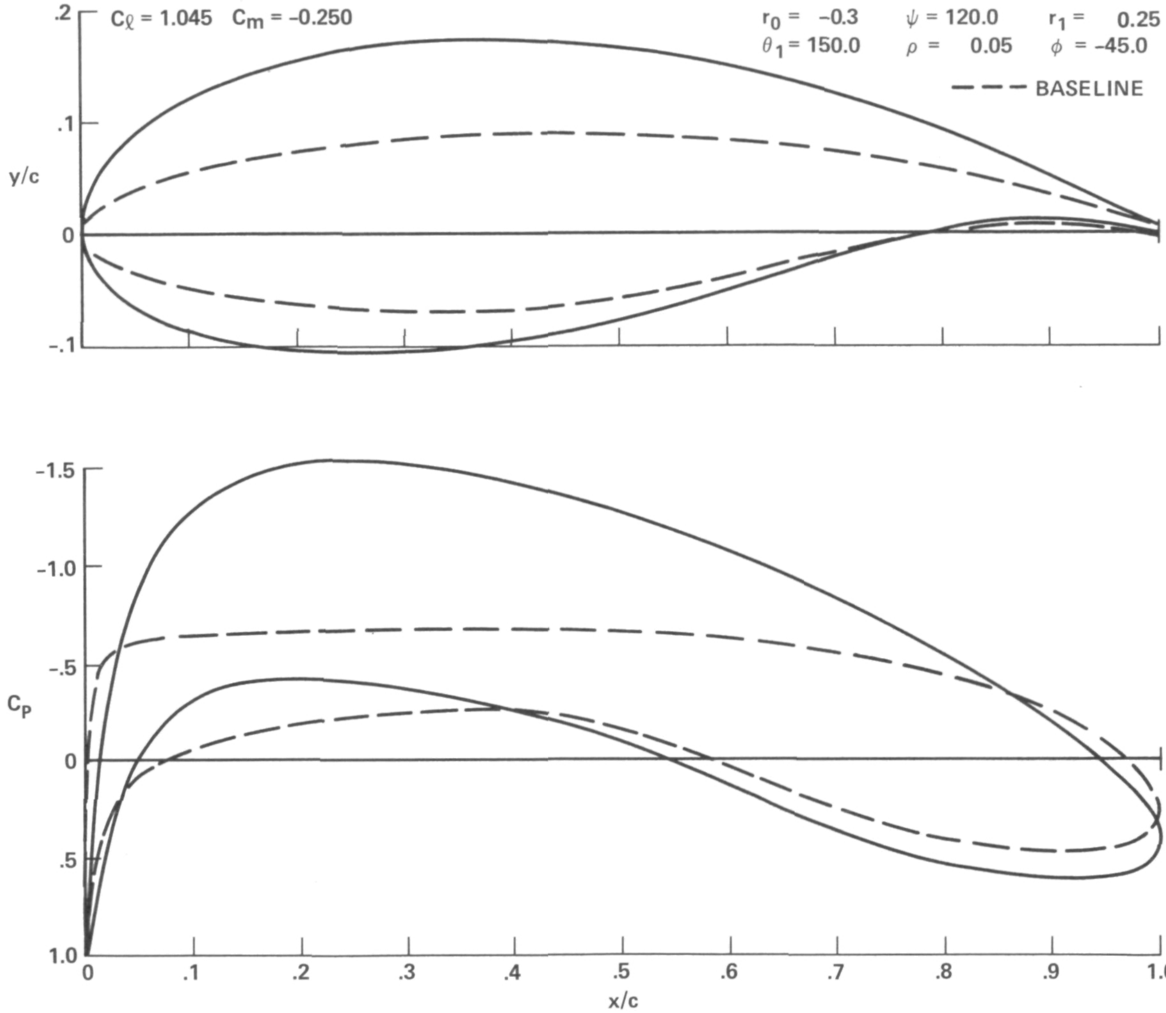


Figure 5.– Profile and aerodynamic changes due to r_1 perturbation, $\alpha = 0°$.

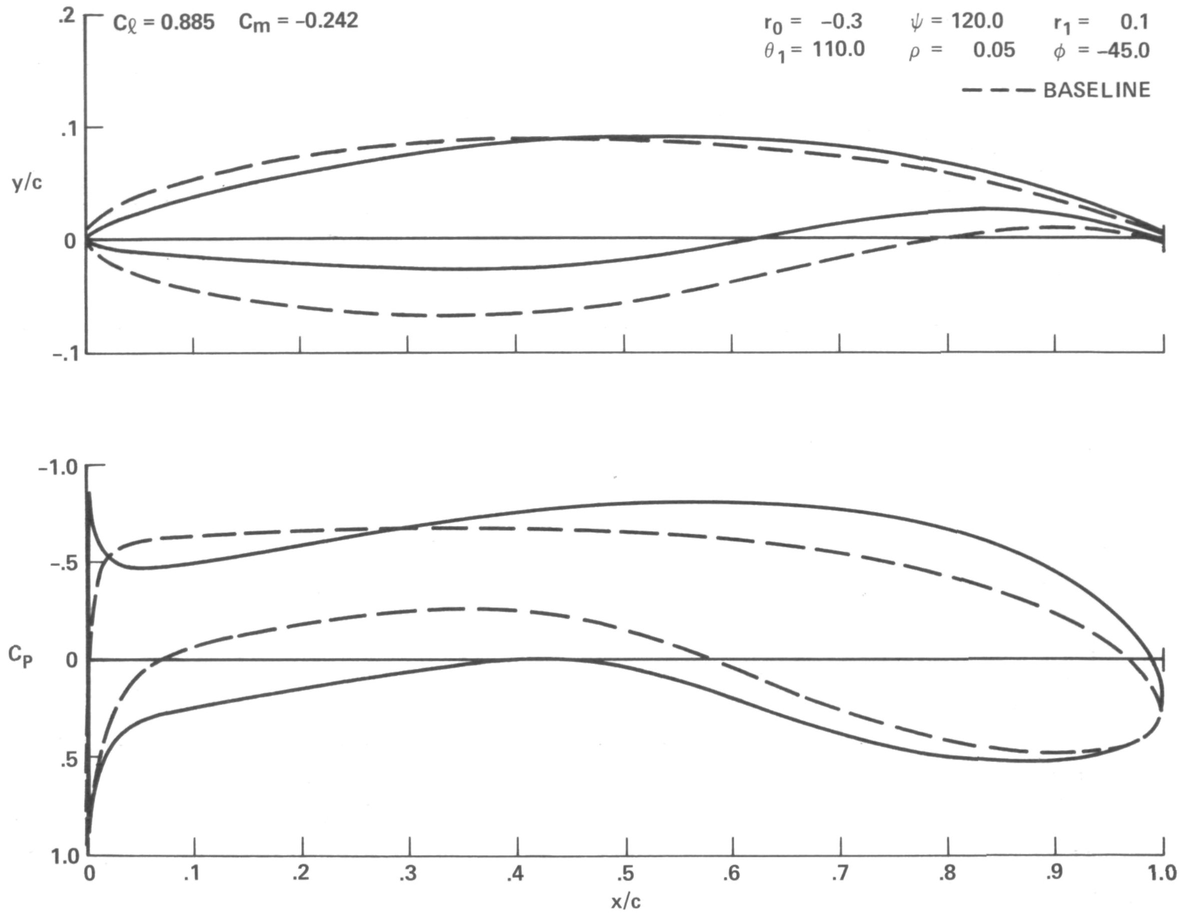


Figure 6.– Profile and aerodynamic changes due to θ_1 perturbation, $\alpha = 0°$.

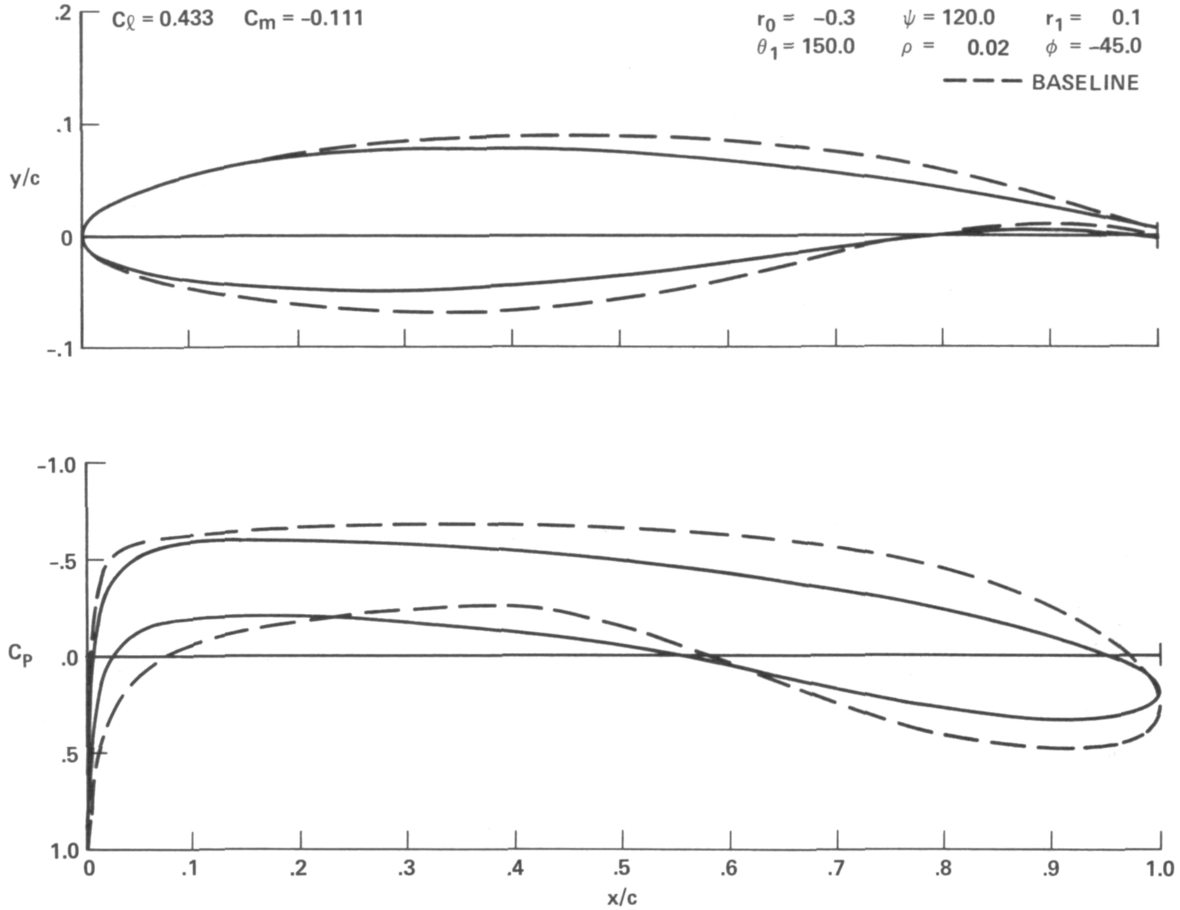


Figure 7.– Profile and aerodynamic changes due to ρ perturbation, $\alpha = 0°$.

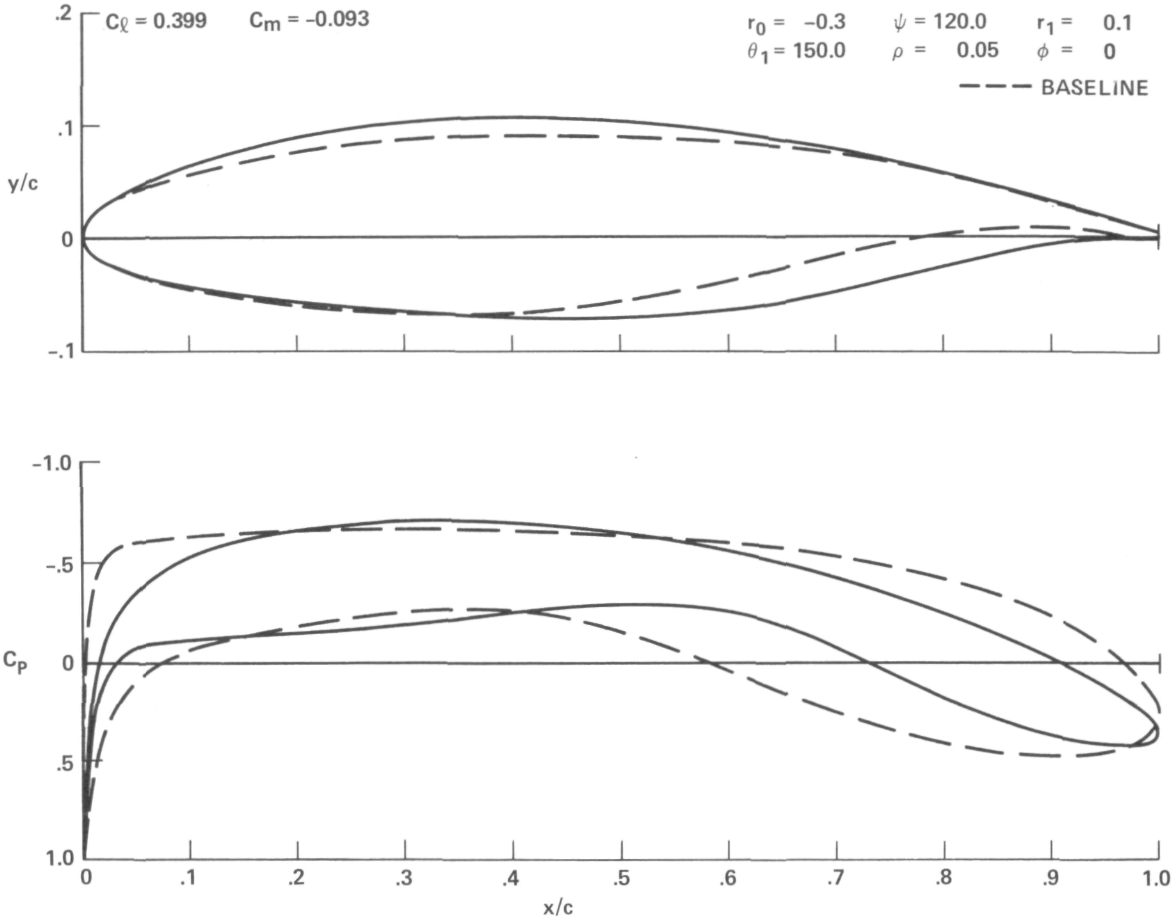


Figure 8.— Profile and aerodynamic changes due to ϕ perturbation, $\alpha = 0°$.

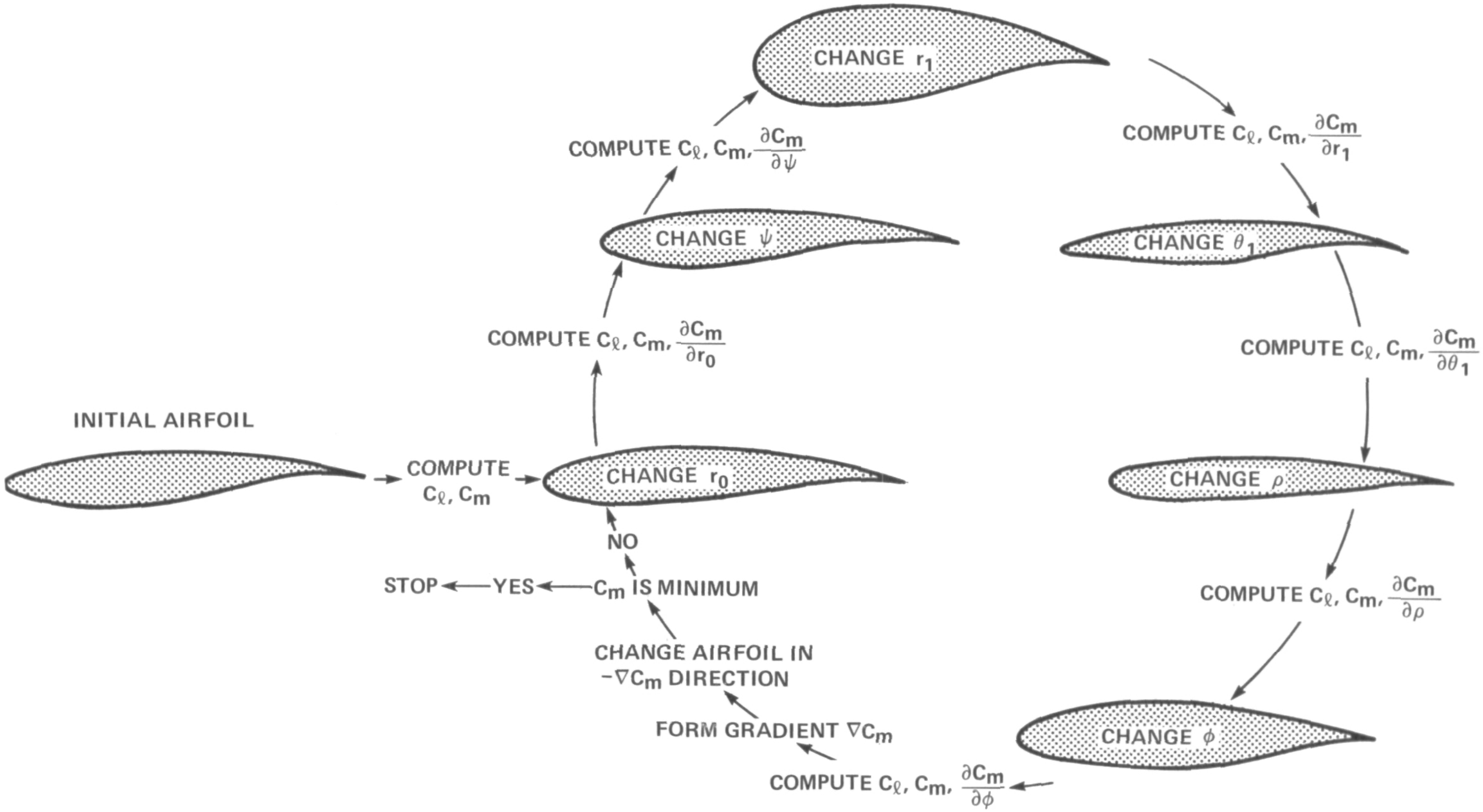


Figure 9.– Optimization flow chart, pitching-moment minimization.

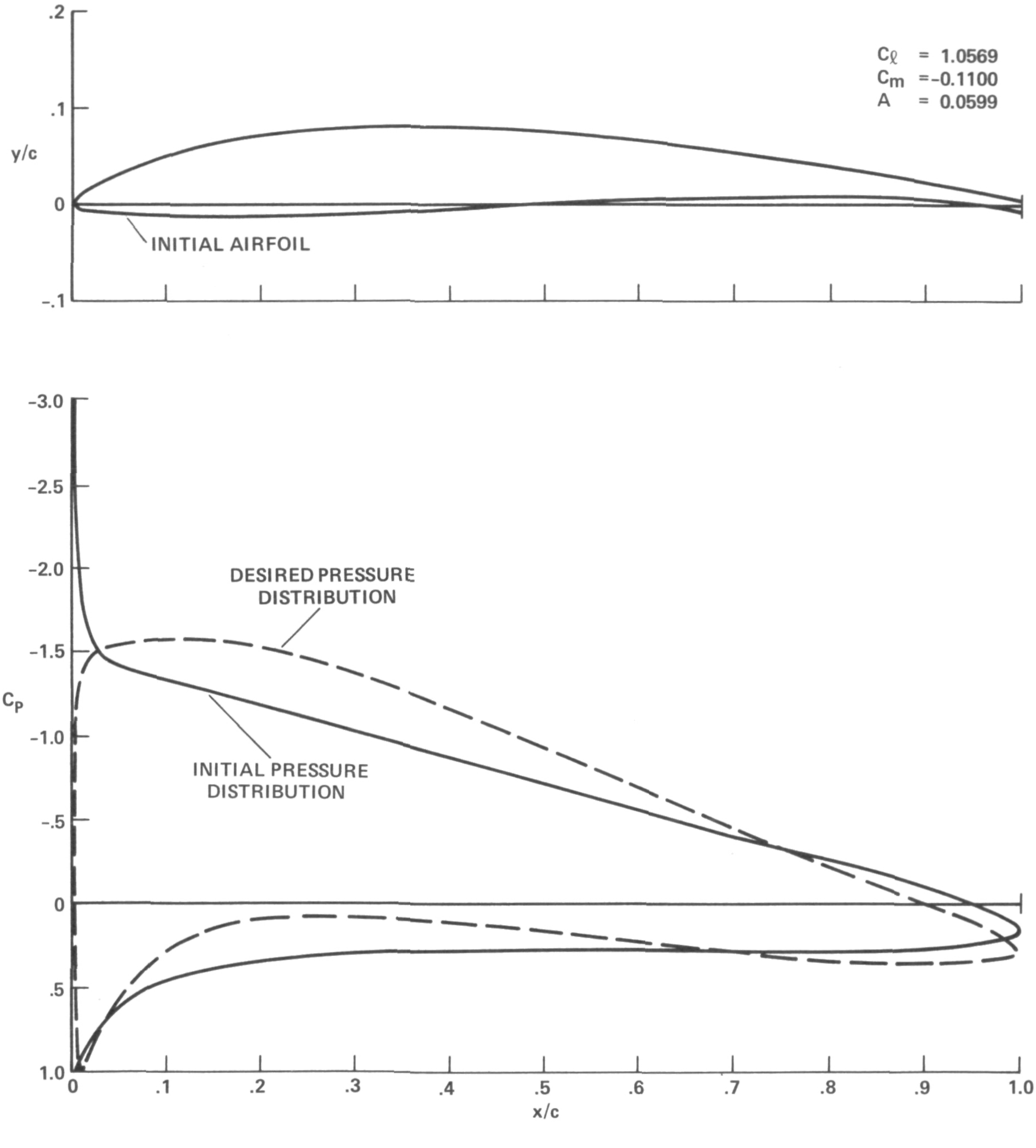


Figure 10.– Pressure distribution design, initial contour and pressure distribution, $\alpha = 5°$.

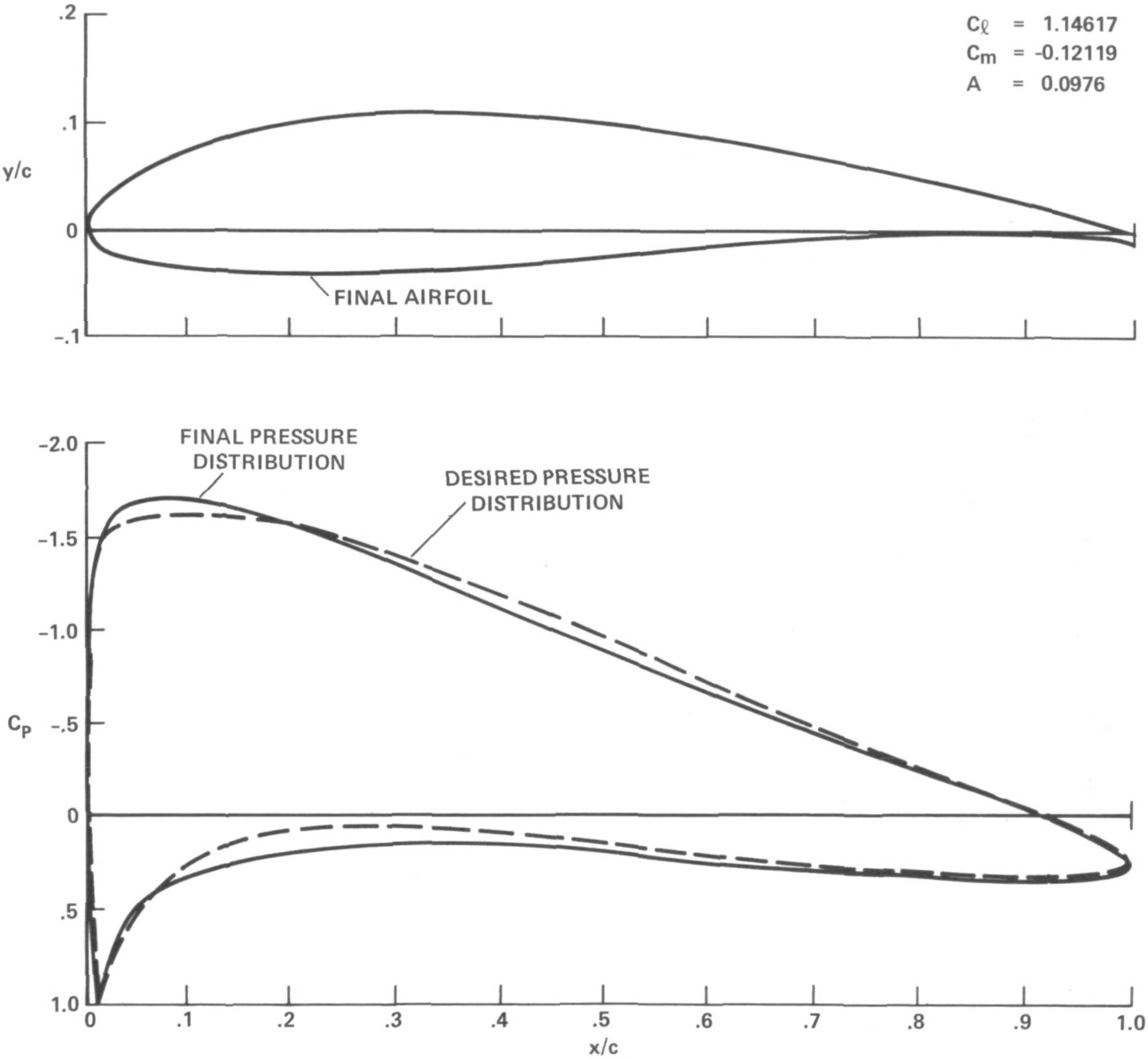


Figure 11.– Pressure distribution design, final contour and pressure distribution, $\alpha = 5°$.

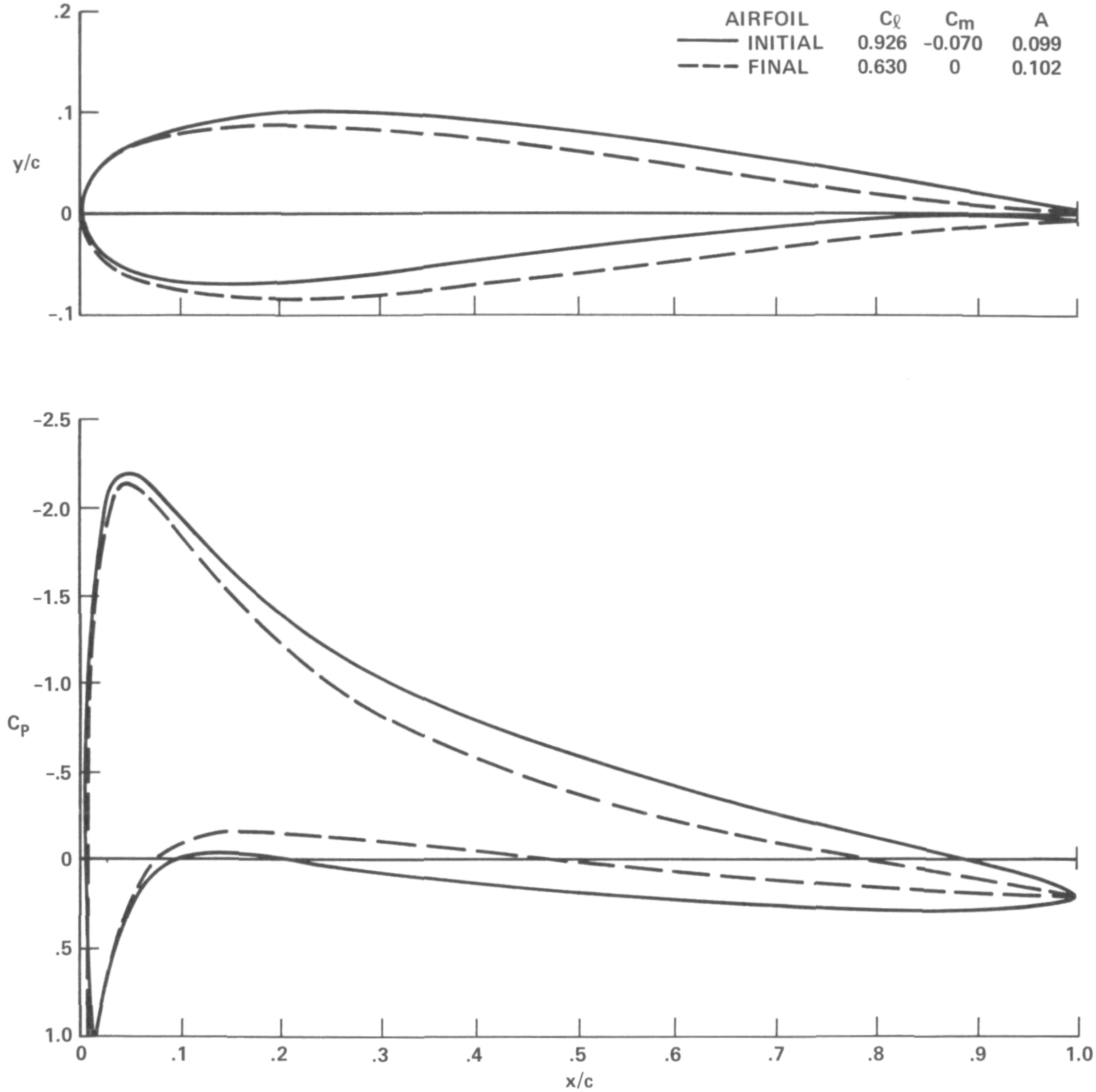


Figure 12.– Unconstrained pitching-moment minimization, $\alpha = 5°$.

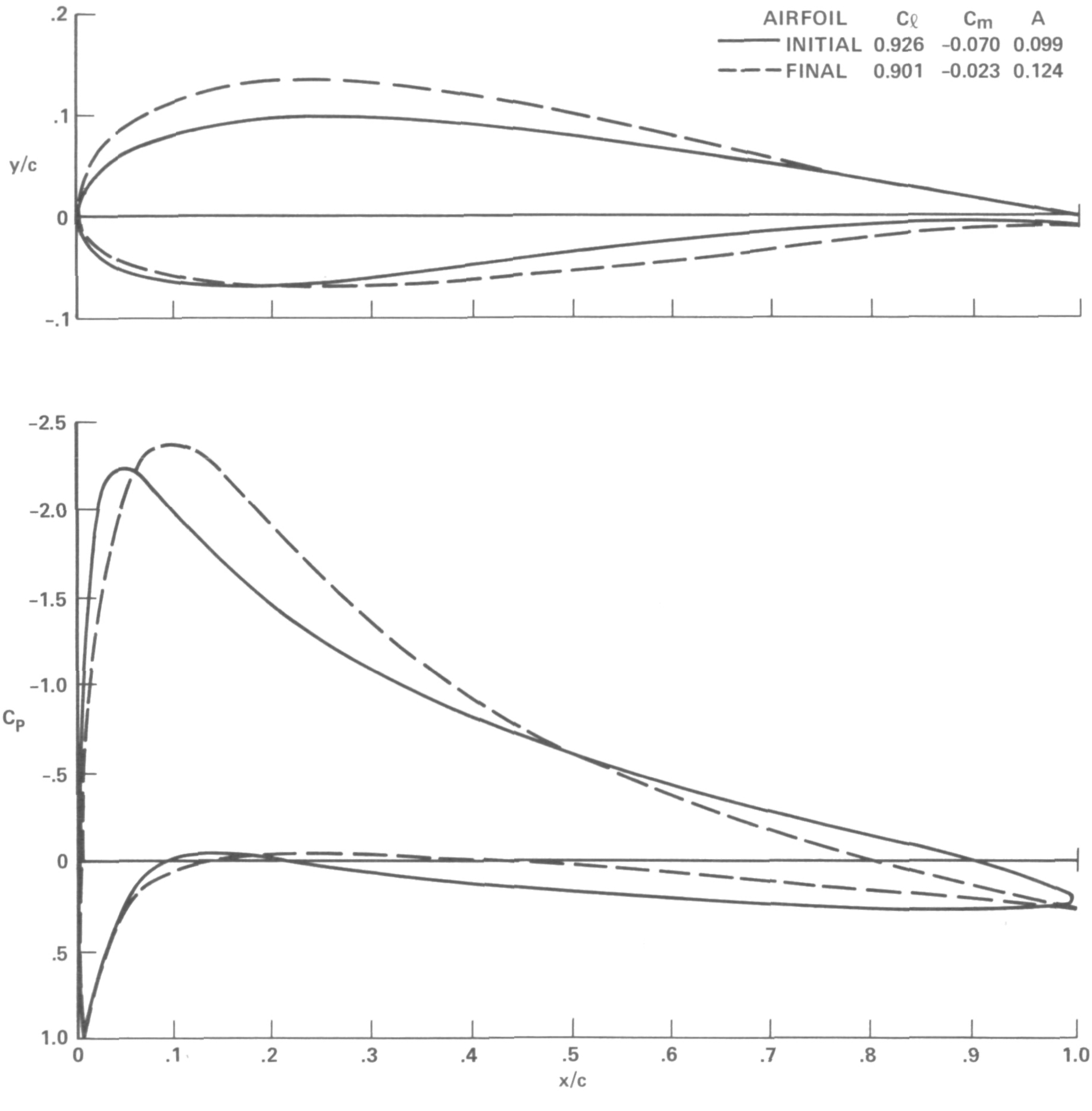
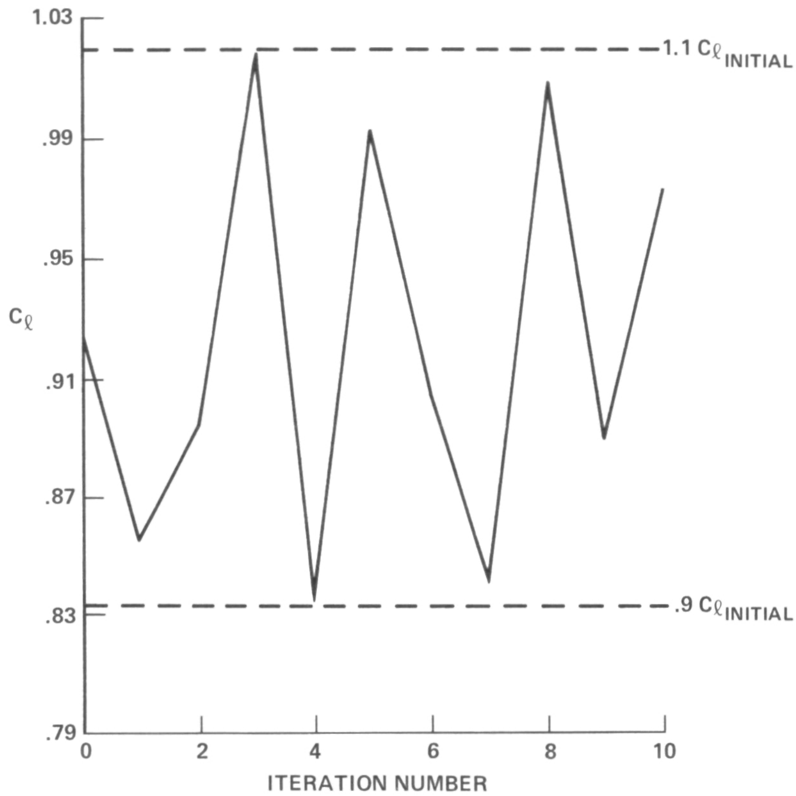
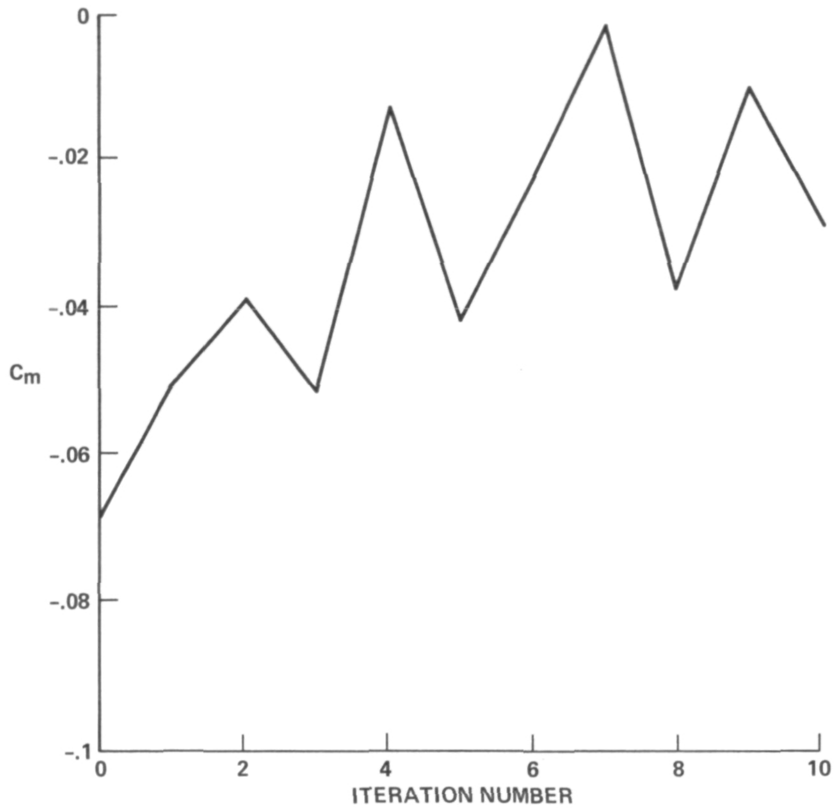


Figure 13.– Pitching-moment minimization with c_l constraint, $\alpha = 5°$.



(a) Constraint history.

Figure 14.– Pitching-moment minimization with c_l constraint.



(b) Optimization history.

Figure 14.– Concluded.

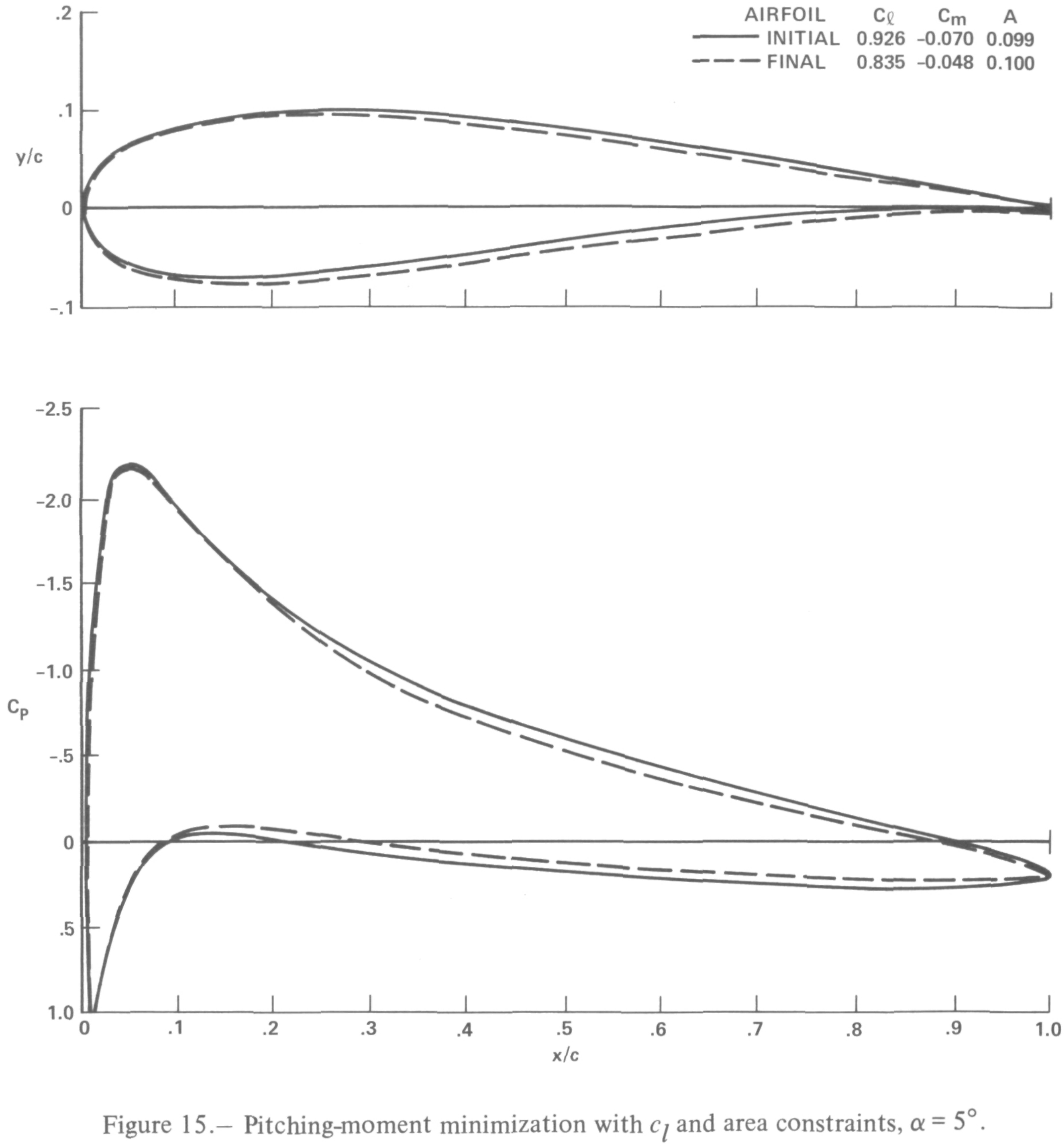


Figure 15.– Pitching-moment minimization with c_l and area constraints, $\alpha = 5°$.

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16. Abstract

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